Topology optimization for a trade off between energy cost and network lifetime in average consensus

{Cesar.Asensio, Daniel.Alonso, Fernando.Camaro, Baltasar.Beferull}@uv.es

IRTIC, UNIVERSIDAD DE VALENCIA, SPAIN

Group of Information and Communication Systems

Summary of Contributions

New topology optimization method to:

1) Reduce the convergence time

2) Reduce the total energy consumption

3) Increase network lifetime

Based on multi-objective minimization.

Background

- Set of nodes and edges: $\mathbf{V}$ and $\mathbf{E}$.

- Communication flow: $\mathbf{G} = (\mathbf{V}, \mathbf{E})$.

- Adjacency matrix: $\mathbf{A}$ such that $[\mathbf{A}]_{ij}$ is equal to 1 if $(i,j) \in \mathbf{E}$ and 0 otherwise.

- Degree matrix: $\mathbf{D}$ diagonal matrix, whose entries are given by $d_i = \sum_{j=1}^{N} \mathbf{A}_{ij}$.

- Laplacian matrix: $\mathbf{L} = \mathbf{D} - \mathbf{A}$.

System evolution

- Initial data at time instant $t = 0$: $\mathbf{x}(0)$, whose average is $\mathbf{x}_{\text{avg}} = \frac{\mathbf{1}\mathbf{1}^T\mathbf{x}(0)}{N}$.

General linear update of sensors at time instant t:

$$\dot{\mathbf{x}}(t) = -\mathbf{L}\mathbf{x}(t)$$

- Convergence time T_{conv}: time required by slowest mode to be reduced by a factor $\rho < 1$

- Expressed as $T_{\text{conv}} = -T_s \frac{\log(\rho)}{\lambda_2(\mathbf{L})}$, where T_s is the duration of a time slot unit and $\lambda_2(\mathbf{L})$ is the so-called algebraic connectivity.

Multi-objective function

Convergence time: T_{conv}.

Total energy consumption:

$$\mathcal{E}_{\text{tot}} = T_{\text{conv}} P_{\text{iter}} = T_{\text{conv}} \sum_{i \in \mathbf{V}} P_i = T_{\text{conv}} \sum_{(i,j)\in\mathbf{E}} p_{ij}$$

Network lifetime proportional to:

$$\mathcal{L} = \frac{\mathcal{E}_{\text{node}}}{T_{\text{conv}} P_{i_{\max}}}$$

Numerical results

Table 1. Simulation Results with $a_{\text{th}} = 0.45$

Graph	$\mathcal{L}^{\beta=1}$	$\mathcal{E}_{\text{tot}}^{\beta=1}$	$\mathcal{L}^{\beta=0}$	$\mathcal{E}_{\text{tot}}^{\beta=0}$	$\mathcal{L}^{\beta=\frac{1}{2}}$	$\mathcal{E}_{\text{tot}}^{\beta=\frac{1}{2}}$
none	795	0.037	475	0.035	734	0.036
RGG	1000	0.038	600	0.027	750	0.027
SWG	1100	0.031	440	0.024	850	0.025

The proposed algorithm: Projection is divided in several consensus process

Optimization problem (non convex) **P1**:

$$\begin{aligned} \text{minimize}_{\{\mathbf{A}\}} \quad & \beta \frac{P_{i_{\max}}(\mathbf{A})}{\lambda_2(\mathbf{L}(\mathbf{A}))} + (1-\beta)\frac{P_{\text{iter}}(\mathbf{A})}{\lambda_2(\mathbf{L}(\mathbf{A}))} \\ \text{s. t.} \quad & \xi \leq \lambda_2(\mathbf{L}(\mathbf{A})) \\ & \mathbf{A} = \mathbf{A}^T \\ & [\mathbf{A}]_{ij} \in \{0,1\} \end{aligned}$$

- ξ: arbitrary small positive constant to ensure resulting value of $\lambda_2(\mathbf{L}(\mathbf{A})$ greater than zero.

- β: constant between 0 and 1 that controls trade off between total energy consumption and network lifetime.

Relaxed optimization problem (convex) **P2**:

$$\begin{aligned} \min._{\{s,\mathbf{A}\}} \quad & s \\ \text{s. t.} \quad & \beta P_1(\mathbf{A}) + (1-\beta)P_{\text{iter}}(\mathbf{A}) - \mu\lambda_2(\mathbf{L}(\mathbf{A})) \leq s \\ & \vdots \\ & \beta P_N(\mathbf{A}) + (1-\beta)P_{\text{iter}}(\mathbf{A}) - \mu\lambda_2(\mathbf{L}(\mathbf{A})) \leq s \\ & \xi \leq \lambda_2(\mathbf{L}(\mathbf{A})) \\ & \mathbf{A} = \mathbf{A}^T \\ & 0 \leq [\mathbf{A}]_{ij} \leq 1 \end{aligned}$$

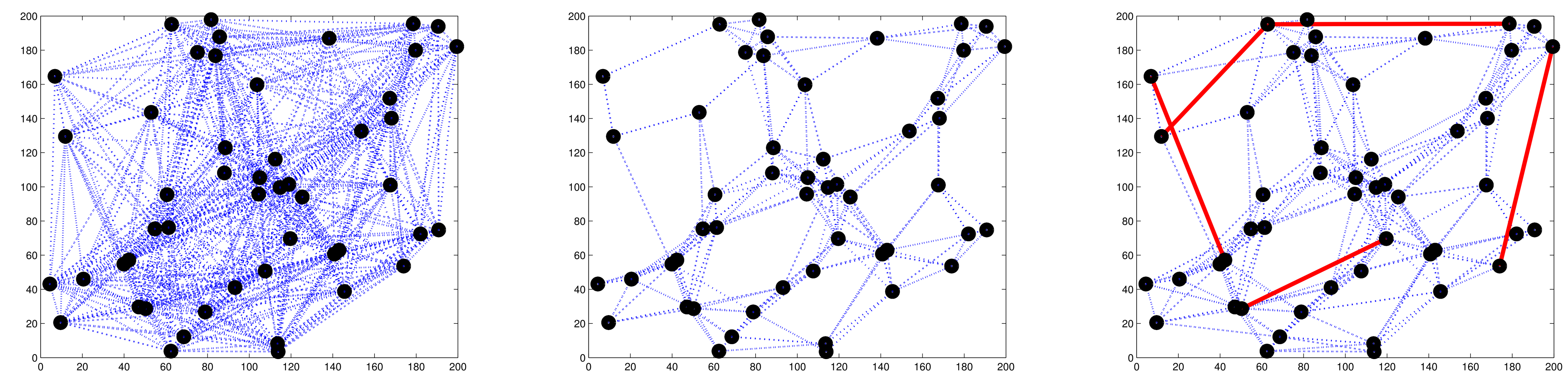
P2 is the relaxed epigraph form of the original problem **P1**.

Dinkelbach's algorithm:

Set matrix $\mathbf{A}$ as a feasible solution
WHILE $(\beta P_{i_{\max}}(\mathbf{A}) + (1-\beta)P_{\text{iter}}(\mathbf{A}) - \mu\lambda_2(\mathbf{L}(\mathbf{A})) > \epsilon)$

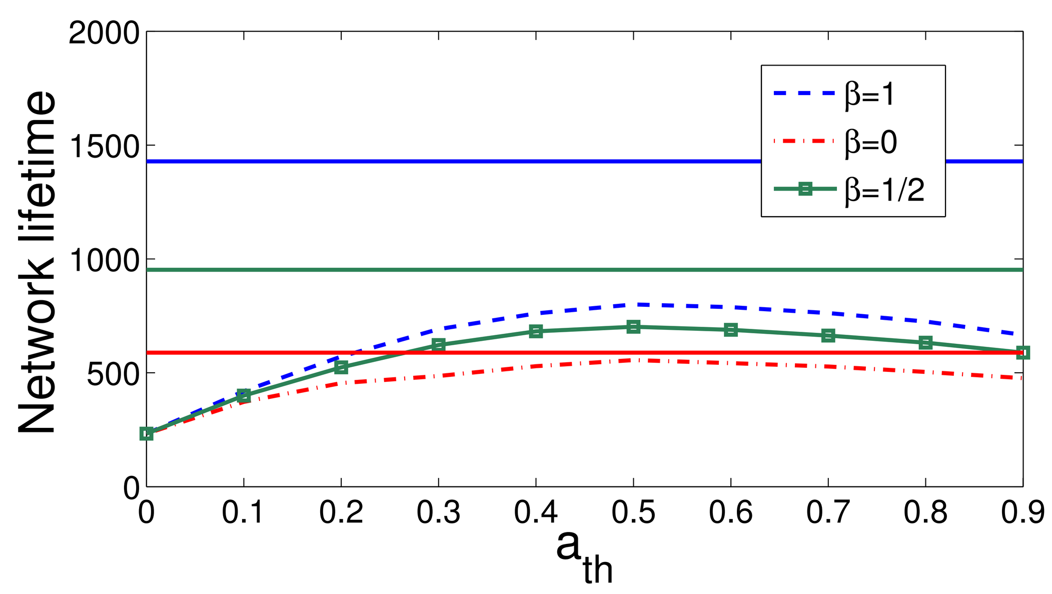
- set μ as $\frac{\beta P_{i_{\max}}(\mathbf{A}) + (1-\beta)P_{\text{iter}}(\mathbf{A})}{\lambda_2(\mathbf{L}(\mathbf{A}))}$
- Solve **P2** with the current μ

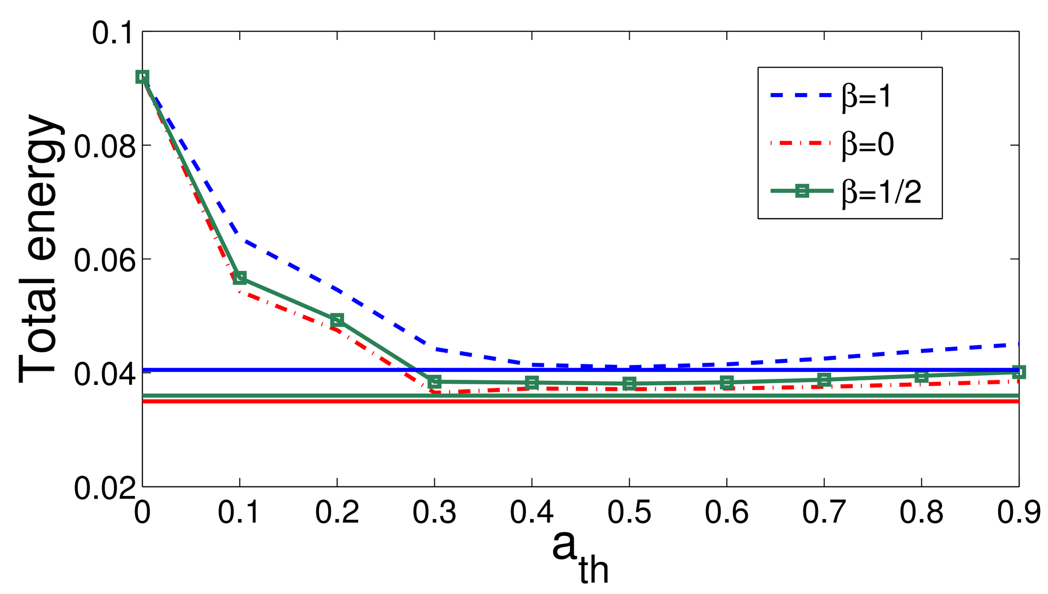
END WHILE



Results: Projecting to the original solution set

Simulation setup: $N = 50$ nodes. Path loss exponent $\gamma = 3$, a signal to noise threshold $\Phi = 10$, and background noise $N_0 = 10^{-10}$ mW., such that $p_{ij} = \Phi N_0 r_{ij}^{\gamma}$ mW. Finally, $\mathcal{E}_{\text{node}} = 1$ and $T_s = 1$ ms.





a) $\beta = 1$: a lifetime maximum is obtained for threshold values around $a_{\text{th}} = 0.45$.
b) $\beta = 0$: minimization of the total required energy.
c) $\beta = 1/2$: Pareto optimal solutions.

Funding

This work was supported by the Spanish MEC Grants TEC2010-19545-C04-04 "COSIMA", CONSOLIDER-INGENIO 2010 CSD2008-00010 "COMONSENS", the European STREP Project "HYDROBIONETS" Grant no. 287613 within the FP 7 and by a Telefonica Chair.