# Fast Average Gossiping Under Asymmetric Links in Wireless Sensor Networks

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## Contributions

**Proposal:** A new algorithm for average consensus:

1) Efficient in terms of consensus time
2) Asymmetric communications allowed
3) Possibility of being performed distributively

**Result:** No control mechanism are necessary.

## Formulation: Graph theory

- Network modeled as graph $\mathbf{G}(k) = (\mathbf{V}, \mathbf{E}(k))$, set $\mathbf{V}$ of $N$ nodes and set $\mathbf{E}$ of edges.

- Neighbors $i$: $\mathbf{\Omega}_i(k) = \{j \in \mathbf{V} : (i,j) \in \mathbf{E}(k)\}$.

- Adjacency matrix $\mathbf{A}(k)$, where an entry $[\mathbf{A}(k)]_{ij}$ is equal to 1 if $(i,j) \in \mathbf{E}(k)$ and 0 otherwise.

- Laplacian matrix: $\mathbf{L}(k) = \mathbf{D}(k) - \mathbf{A}(k)$.

- Directed edge: $e_{ij}$, data flow from $i$ to $j$.

## Formulation: Traditional Gossip algorithms

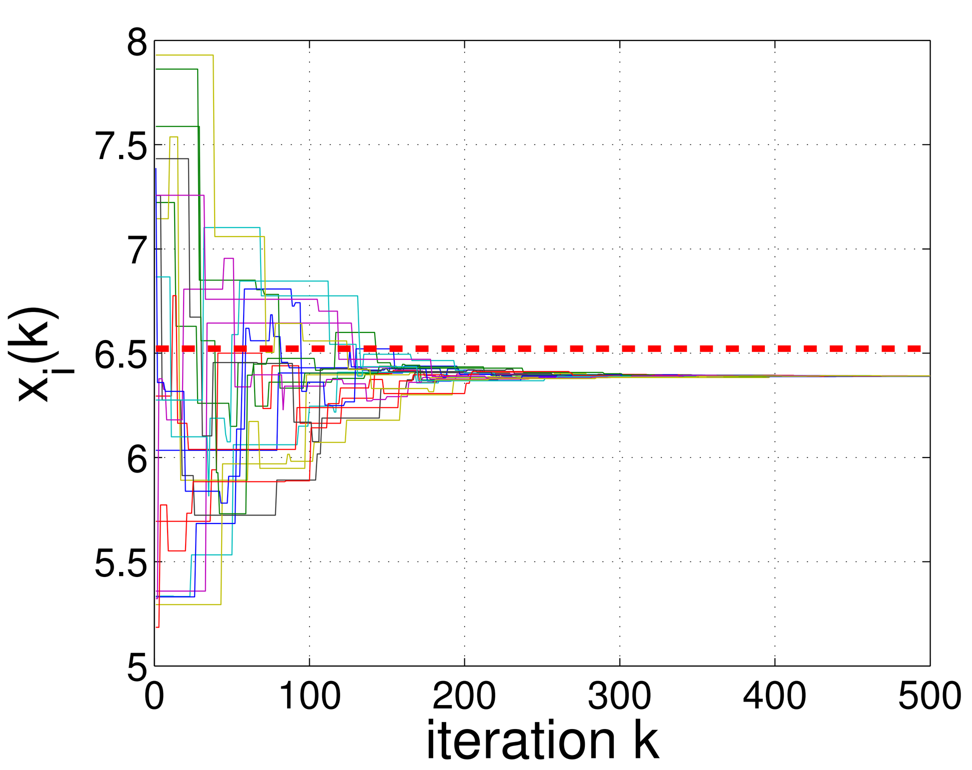
- Initial vector: $\mathbf{x}(0)$, whose average is: $\mathbf{x}_{\text{avg}} = \frac{\mathbf{1}\mathbf{1}^T\mathbf{x}(0)}{N}$, where $\mathbf{1}$ denotes all ones column vector.

- Linear update of sensors state:

$$x_i(k+1) = (1-\epsilon)x_i(k) + \epsilon x_j(k) \quad (1)$$
$$x_j(k+1) = \epsilon x_i(k) + (1-\epsilon)x_j(k) \quad (2)$$

Requisites:
i) Connected undirected graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$
ii) Symmetric $N \times N$ matrix $\mathbf{W}(k)$, at each iteration $k$, such that $[\mathbf{W}(k)]_{ij} = 0$ if $[\mathbf{A}(k)]_{ij} = 0$
iii) $\mathbf{W}(k)\mathbf{1} = \mathbf{1}$, $\mathbf{1}^T\mathbf{W}(k) = \mathbf{1}^T$

## Deviation under asymmetry: (1) but not (2)

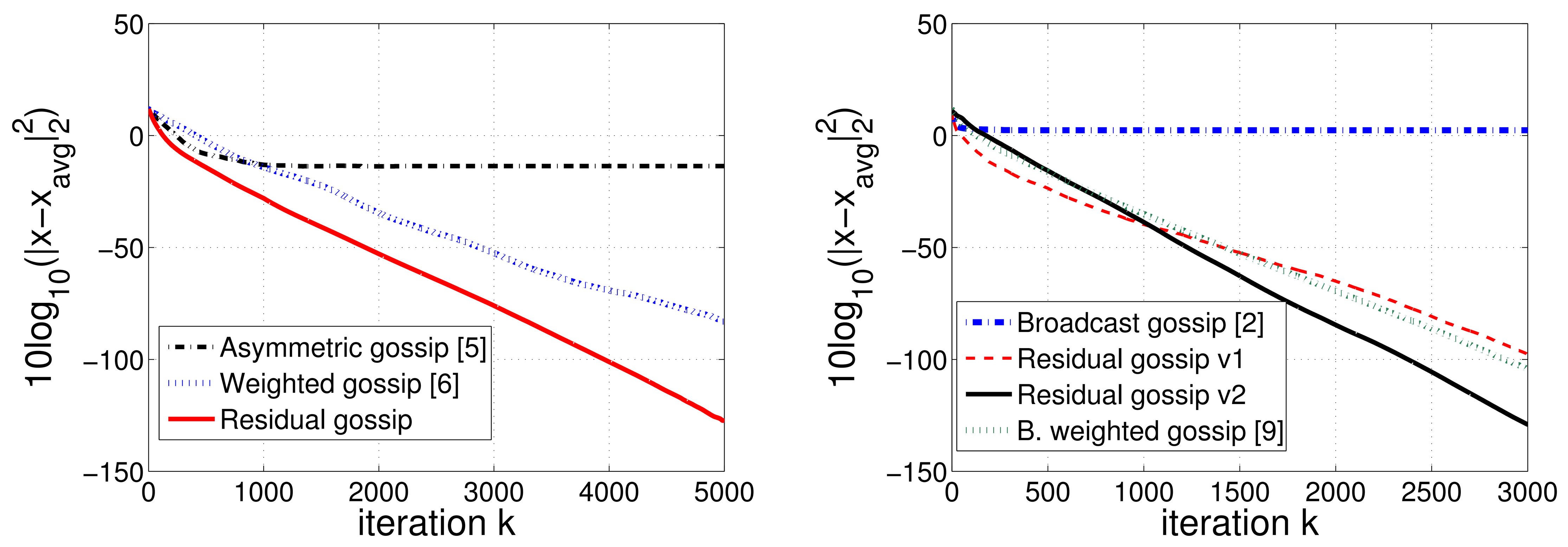



## Cause of the deviation

A residual component is ignored:

$$r_j = x_j(k) - (\epsilon x_i(k) + (1-\epsilon)x_j(k)), \ \forall j \in \mathbf{\Omega}_i(k) \quad (3)$$

## Numerical results




## Proposed methodology

We propose to include in the packet the accumulated residual $r_i$. Thus, system evolves:

$$x_j(k+1) = \epsilon x_i(k) + (1-\epsilon)(x_j(k) + r_i(k)), \ \forall j \in \mathbf{\Omega}_i(k) \quad (4)$$

which implies that the residuals evolve as follows:

$$r_j(k+1) = r_j(k) + x_j(k) - x_j(k+1) + r_i(k), \ \forall j \in \mathbf{\Omega}_i(k) \quad (5)$$

and $r_i(k+1) = 0$.

Scheme defined by (4) and (5) preserves the summation of the initial data along iterations. Is still able to converge to a common state? If does, it is surely the average.

From expression (4), the weight matrix $\mathbf{W}(k)$ of the asymmetric gossip is:

$$[\mathbf{W}(k)]_{\ell j} = \begin{cases} 1-\epsilon & \text{if } j = \ell, \ \ell \in \mathbf{\Omega}_i(k) \\ \epsilon & \text{if } j \neq \ell, \ j = i, \ \ell \in \mathbf{\Omega}_i(k) \\ 1 & \text{if } j = \ell, \ \ell \notin \mathbf{\Omega}_i(k) \\ 0 & \text{otherwise} \end{cases}$$

where node $i$ is chosen to send data in the current iteration $k$.

Equivalently, we define the residual vector $\mathbf{s}$ as follows:

$$[\mathbf{s}(k)]_j = \begin{cases} r_i(k) & \text{if } j \in \mathbf{\Omega}_i(k) \\ 0 & \text{otherwise} \end{cases}$$

which allows us to express (4) as:

$$\mathbf{x}(k+1) = \prod_{i=0}^{k}(\mathbf{W}(i)\mathbf{x}(0)) + \sum_{j=0}^{k}\left(\prod_{\ell=j}^{k}(\mathbf{W}(\ell)\mathbf{s}(j))\right) \quad (6)$$

**Proposition 1:** The system defined by (6) reaches probabilistic average consensus $\lim_{k\to\infty} \mathbf{x}(k) = \mathbf{x}_{\text{avg}}$, provided that $\lim_{k\to\infty} r_i(k) = 0 \ \forall i \in \mathbf{V}$.

**Methodology to exploit residuals (unicast):**

*Min-max approach* (h1): Choose neighbor with minimum state value if residual of origin node $i$ is positive and or the maximum if it is negative.

**Methodology to exploit residuals (broadcast):**

*Equitable approach* (h2): Equally distributing the residuals. The origin node $i$ broadcastly sends $(x_i(k), \frac{r_i(k)}{d_i})$ to every node $j \in \mathbf{\Omega}_i(k)$, that computes its new state (4) and residual (5).

*Large-small approach* (h3): Update neighbors with smaller state value than node $i$ if current residual is positive or it updates the state of the neighbors with larger state value otherwise. The rest of neighboring nodes only update their residual.

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